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DYNAMIC ANALYSIS OF ELEVATION INDICATOR MODULE, XM224 60 mm MORTAR

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Frankford Arsenal
Philadelphia, Pennsylvania

October 1973

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This report describes a dynamic analysis of the elevation endicator module on the XM224 mortar which was conducted to determine the cause of level vial failures during field firing. The unit is modeled as a spring mounted beam and its response to a theoretical firing shock is determined. The analysis shows that the module's lack of symmetry results in a severe rotational response for a purely translational shock input. As a result of rotational response, the level vial experiences relatively high bending stresses during the firing shock. It is concluded that these high bending stresses are the cause of the field failures. Recommendations are made for increasing the shock resistance of the module by increasing its bending stiffness and by making the unit more symmetrical.

Details of illustrations in this document may be better studied on microfiche.

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by

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Technical Support Directorate
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Philadelphia, PA 19137

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ABSTRACT

This report describes a dynamic analysis of the elevation indicator module on the XM224 mortar which was conducted to determine the cause of level vial failures during field firing. The unit is modeled as a spring mounted beam and its response to a theoretical firing shock is determined. The analysis shows that the module's lack of symmetry results in a severe rotational response for a purely translational shock input. As a result of rotational response, the level vial experiences relatively high bending stresses during the firing shock. It is concluded that these high bending stresses are the cause of the field failures. Recommendations are made for increasing the shock resistance of the module by increasing its bending stiffness and by making the unit more symmetrical.

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INTRODUCTION

The Elevation Indicator Module is part of the fire control system on the 81 mm mortar and permits quick deployment and firing of the weapon in a hand held mode. This capability is useful when the tactical situation does not permit setup of the complete weapon and use of the more complex sight unit. The indicator, consisting of a level vial and scale, provides an indication of the required weapon elevation based on the estimated distance to the target. The module is incorporated into the handle of the 81 mm Mortar and has been used successfully in this application (Figures 1 and 2).

When the indicator was employed on the development model of the XM224, 60 mm mortar however, breakage of the level vial occurred during firing of the weapon at maximum charge. In each case, the level vial cracked at the end nearest the mortar tube. This report will describe a dynamic analysis which was performed to determine the cause of the failures. Since the only hardware available was the handle assembly for the 81 mm mortar, the analysis is based primarily on the characteristics of this assembly. The units for the two mortars are very similar, however, and the analysis is considered to be sufficiently valid to identify the general characteristics of the system's response to the firing shock.

DESCRIPTION OF THE PHYSICAL SYSTEM

The level vial and scale are attached to a handle on the mortar by means of RTV adhesive sealant as shown below.

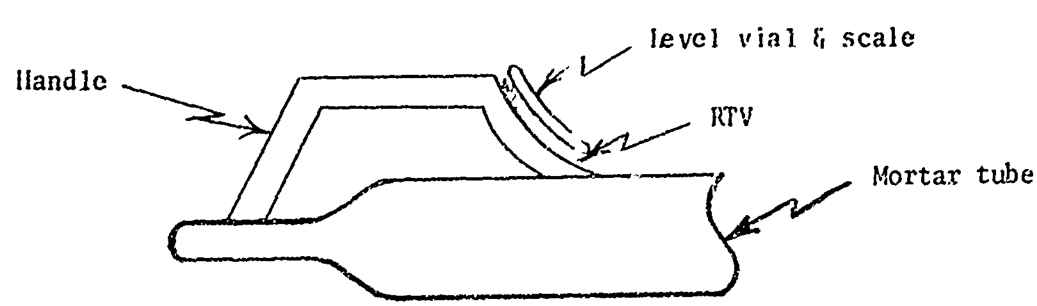


Figure 1. Elevation Indicator Module - 81 mm Mortar (Assembled View)

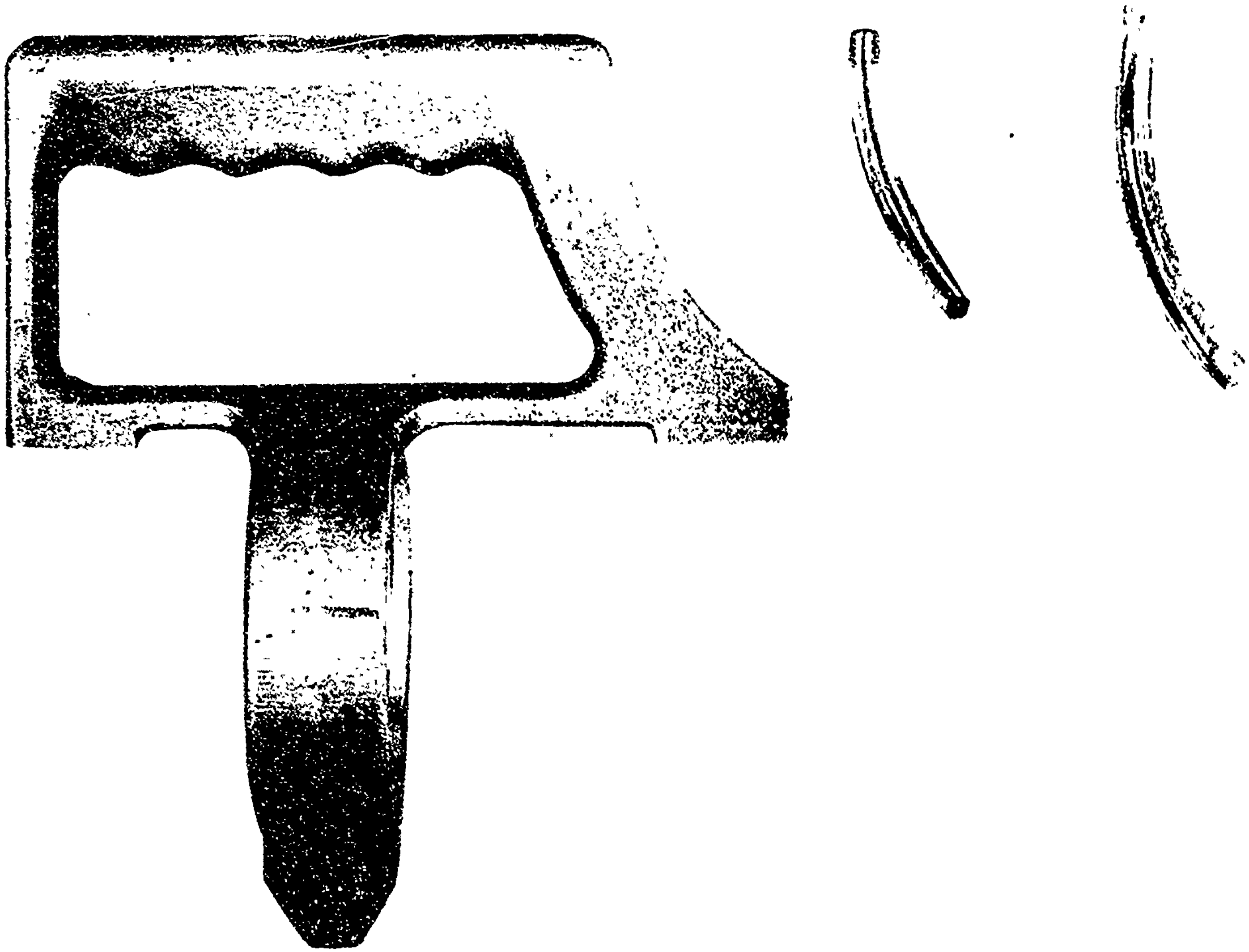


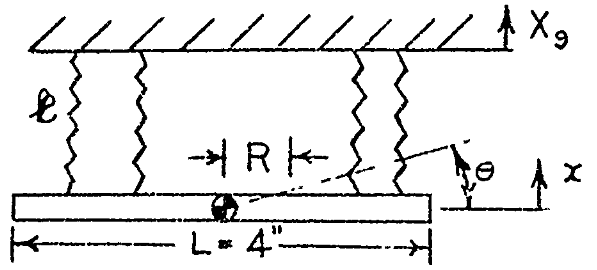
Figure 2. Elevation Indicator Module - 81 mm Mortar (Exploded View)

Examination of the assembly revealed the essential characteristics which could be used in constructing a mathematical model. The RTV cushion is very much more flexible than either the level vial and scale or the handle. This means that the vial and scale are essentially rigid relative to the stiffness of the RTV. It was also found that the module was more flexible on the upper end than on the end nearest the mortar tube. From this it can be concluded that the system is not symmetrical, and the resultant spring force of the RTV cushion does not act through the c.g. of the system.

As a result of this unsymmetrical system, firing of the weapon causing a purely rearward movement of the handle, would produce both translation and rotation of the vial and scale. Rotation of the level vial, opposed by the RTV cushion, would induce bending stresses in the vial which could account for the field failures. The following analysis will investigate this phenomena to determine if the resulting stresses are high enough to cause failure of the vial.

MATHEMATICAL MODEL OF SYSTEM

The elevation indicator module will be modeled as a beam (the vial and scale), attached to a foundation (the handle) by elastic springs (the RTV), and excited by a movement Xg(t) of the foundation. Shown below is a sketch of the model.



Due to the lack of symmetry of the system the resultant line of action of the spring forces is not through the c.g. but acts at some distance (R) from the c.g. The cushion material is assumed to have a resultant spring constant k_x for translation and k_θ for rotation. The equations of motion for the system are as follows where $\ddot{x}$ and $\ddot{\theta}$ are the linear and angular acceleration, m is the mass,

and J is the mass moment of inertia.

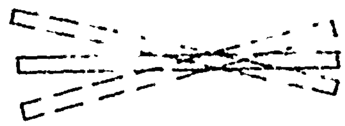
$$mx + k_x (x + R\theta) = k_x Xg$$

$$J\theta + (k_\theta + k_x R^2)\,\theta + k_x RXg$$

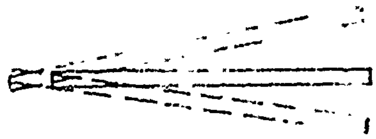
These equations are solved in detail in the Appendix, and only a summary of the analysis will be given here in order that the main thrust of the argument is not obscured by the mathematics. Suffice to say that values for the various physical parameters of the module were measured or estimated, the natural frequencies and mode shapes of the system were determined, and finally the response of the module to an assumed motion of the mortar was calculated.

The natural frequencies of the system were calculated to be at 540 cps and 780 cps, with the corresponding mode shapes being as shown below.

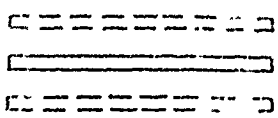
$f_n = 540$ cps



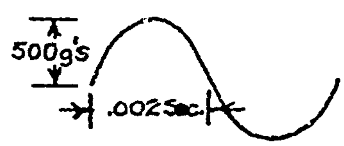
$f_n = 780$ cps



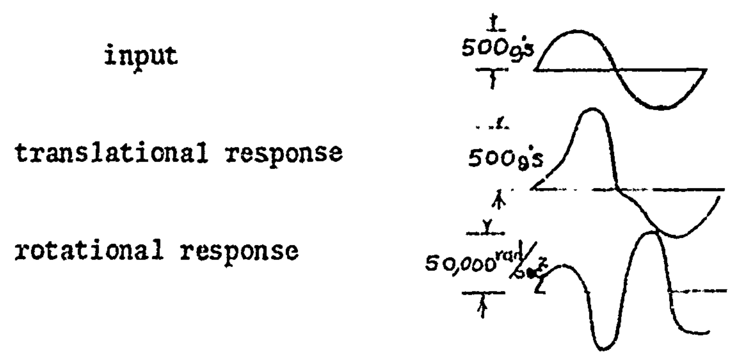
The first mode was observed during laboratory vibration at 560 cps, with the mode shape being essentially as calculated except that the node was closer to the right end of the vial. These mode shapes are the result of coupling between the translational and rotational modes of the system, or in other words, due to the module's lack of symmetry. Thus a purely translational input will result in both translation and rotation of the unit. If the system were symmetrical, a translational input would result in a purely translational response. This was also shown in the laboratory where some of the RTV was removed (from one end of the module) so that the cushion material was more evenly distributed. The response then to the input of a pure translation was also a pure translation as shown below.



The response of the system was then determined for an assumed sinusoidal acceleration of the mortar as shown below:



The 500g level seem reasonable in light of the internal ballistics data in Reference 1. The full sine wave for acceleration was chosen so that the mortar velocity at the end of the shock would be zero, although there would be a net displacement. No actual firing data has come to my attention, but it is believed the parameters chosen here are sufficiently representative of the mortar firing shock to determine the general characteristics of the module's response. The calculated response time histories for both the translational and the angular acceleration are shown below, along with the input acceleration time history.



The theoretical response curves show that the module is subjected to severe rotational acceleration even though the assumed input motion is a pure translation. This type of response could be anticipated based on the previous discussion. A possibly more vivid way of showing the module's response is given below indicating its position (relative to the mortar) at several points in time.

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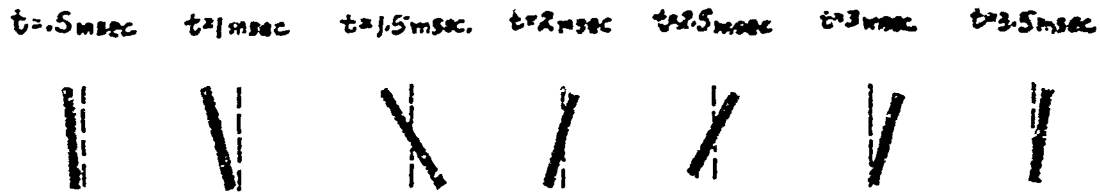
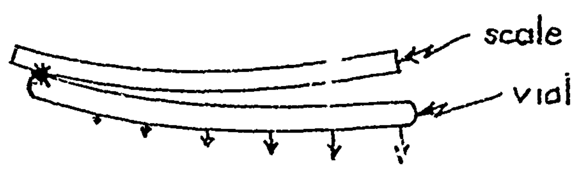
[1] L. Heppner, "Special Study of Setback and Spin for Artillery and Tank Ammunition," April 1966, Aberdeen Proving Ground, Report No. DPS-1963.

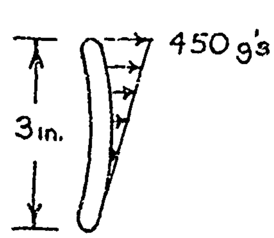


The rotational motion of the indicator module is believed to be very significant in light of the failures which occur at the end of the vial. For pure translational motion, maximum stresses would tend to develop near the center of the vial. During rotational motion of the module, however, at least two phenomena could develop high stresses at the end of the vial. The vial and scale can be expected to execute somewhat different motions, and impacting could occur at the end of the vial if the two pieces move out of phase. A stress concentration at the metal seal in the end of the vial may also contribute to increasing the local stress.

A related possibility is that a cantilever bending mode will develop when one end of the vial tends to be clamped by the scale, and the distributed force due to deflection of the cushion causes bending of the vial as indicated below.



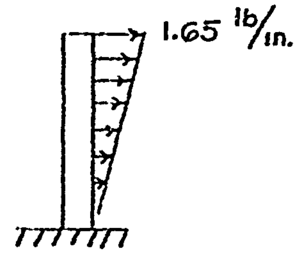
At t = .0016 seconds, for example, the vial is essentially rotating about its end and the total acceleration is approximately as shown:



The maximum value of the inertia load of the vial, which weighs .011 lbs, then is

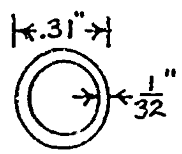
$$w_{max} = \frac{.011}{3}(450) = 1.65\text{-lb/in.}$$

If the cantilever type of bending develops, the maximum moment at the end of the vial would be



$$M_{max} = \frac{1}{2}(1.65)(3)(2) = 4.95 \text{ in.-lb}$$

The cross section and moment of inertia of the level vial are as shown below



$$I = .000273 \text{ in.}^4$$

The maximum bending stress produced by the 4.95 in-lb moment then would be

$$f_b = \frac{Mc}{I} = \frac{4.95\ (.155)}{.000273} = 2800 \text{ lb/in}^2$$

This is a rather high working stress for a brittle material like glass with an allowable stress in the area of 5000 lb/in^2. An expected stress concentration at the metal sealing plug could serve to increase the local stress to a value sufficient to cause failure of the level vial. Even though the full cantilever mode may not develop, it is apparent that the mortar firing shock subjects the level vial to relatively high bending stresses. The vial is weak in bending and this appears to be the most likely cause for the field failures.

Another possible failure mode is when the glass ball impacts the end of the vial causing tension stresses in the vial. This type of phenomena is

difficult to evaluate, but the approximate dynamic load required can be determined to see if this type of failure is likely. The cross sectional area of the vial is .028 in.^{2} and if we use an allowable stress of 5000 lb/in.^{2} to cause tension failure of the vial the force P would be

$$P = f_t\ A = 5000\ (.028) = 140\ \text{lb.}$$

For the glass ball, which weighs about .000265 lb., to exert a dynamic force of this magnitude on the vial, its deceleration would be

$$\text{Deceleration} = \frac{P}{W} = \frac{140}{.000265} = 500,000\text{g's}$$

Even allowing for stress concentrations and glass flaws, ball decelerations of sufficient magnitude to break the glass seem unlikely. Similarly, the deceleration of the column of fluid, weighing about .003 lb, pressing on the end of the vial would have to be about 47,000 g's to cause failure. Again, levels of this magnitude seem very unlikely.

Despite the simple structure of the elevation indicator module, the determination of the dynamic stresses during firing is a very difficult problem. A highly simplified treatment has been used here in an effort to identify the primary failure mode. The field failures occur very quickly, after firing only a few rounds, at maximum charge. This indicates that the problem is not fatigue and not an isolated occurrence, but that the working stress is very high relative to the allowable stress of the glass. Various possible failure modes have been examined, but only a bending phenomena appears to produce stresses high enough to cause failure. It is, therefore, believed that the field failures of the elevation indicator module are due to excessive bending stresses. Lending some credence to this belief is the fact that in at least one instance of failure, the module had pulled almost completely away from the handle, remaining attached only near the tube. This would suggest high forces and deflections at the end of the module away from the tube; that is, the type of rotational response predicted by this analysis.

CONCLUSIONS

It is concluded that the field failures of the level vial in the elevation indicator module are the result of excessive bending stress during firing of the XM224 mortar. The unsymmetrical configuration of the unit results in a severe rotational response even though the input motions may be purely translational. The rotational motion of the module, opposed by the RTV cushion, induces excessive bending stresses in the vial and results in failure of the vial at the end nearest the mortar tube.

RECOMMENDATIONS

To eliminate the failures of the elevation indicator module, it is necessary to increase the bending stiffness o. the level vial. As the simplest method of accomplishing this, it is recommended that the wall thickness of the level vial be incresed to the next standard size, to about .040 in. This approach would require minimal changes in the unit.

Another suggestion was to use an all plastic unit in which the vial and scale are one integral piece A unit of this type would be stiff enough, but care should be taken that the mass is kept small to prevent undue tension stresses in the RTV and possible separation from the weapon. With either approach, effort should be made to make the module symmetrical, with the cushion material being evenly distributed about the center of gravity. This will minimize the rotational response of the unit to the mortar firing shock and the resulting bending stresses.

APPENDIX

Theoretical Determination of Elevation Indicator Module's Response to Shock Excitation

This section contains the detailed analysis of the elevation indicator module's response to a shock excitation.

The equations of motion given in the body of the report are

$$m\ddot{x} + k_x (x + R\theta) = k_x X g$$

$$J\ddot{\theta} + (k_\theta + k_x R^2)\,\theta + k_x = k_x R X g \qquad \text{(A-1)}$$

The solution will be outlined here and is also given in most vibration texts including Reference 2.[2]

In order to determine the natural modes of the system, assume a free vibration condition with $Xg(t) = 0$. To determine the solution to the equation assume

$$x = a_1 \cos pt \qquad \theta = a_2 \cos pt$$

where p is the natural frequency, and substitute into Equation A-1

$$a_1 (-mp^2 + k_x) + a_2 (k_x R) = 0$$

$$a_1 (k_x R) + a_2 (-Jp^2 + k_\theta + k_x R^2) = 0$$

for a non-zero solution, the determinant must be zero.

$$\begin{vmatrix} -mp^2 + k_x & k_x R \\ k_x R & -Jp^2 + k_\theta + k_x R^2 \end{vmatrix} = 0 \qquad \text{(A-2)}$$

The resulting frequency equation is therefore:

$$p^4 - p^2 \left(\frac{k_x}{m} + \frac{k_\theta + k_x R}{J}\right) + \frac{k_x k_\theta}{mJ} = 0 \qquad \text{(A-3)}$$

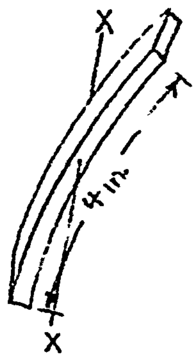
---

[2] Jacobsen & Ayre, "Engineering Vibrations," McGraw-Hill Book Co., 1958.

Equation A-3 can be solved for the two natural frequencies of the system P_I and P_{II}. The corresponding amplitude ratios α_I and α_{II} from Equation A-2 are then

$$\alpha = \frac{a_2}{a_1} = \frac{mp^2 - k_x}{k_x R} \qquad \frac{a_2}{a_1} = \frac{k_x R}{Jp^2 - k_\theta - k_x R^2} \qquad \text{(A-4)}$$

The natural frequencies and mode shapes can now be determined by substituting for the various physical constants of the module, which are estimated as follows. For the level vial and scale:



weight $\omega = .035$ lb.

moment of inertia $J_{xx} \approx \frac{wl^2}{12} = \frac{.035\,(16)}{12} = .047$ lb-in^2

The line of action of the spring forces is assumed to act 1/4 in. from the c.g. therefore

$$R = 1/4 \text{ in.}$$

The translational spring constant is estimated to be approximately 2000 lb/in and the rotational spring constant to be about 1500 in-lb/rad.

thus: $k_x = 2000$ lb/in $\qquad k_\theta = 1500$ in-lb/Rad

Substituting these values into the frequency Equation A-3 and solving:

$P_I = 3370$ rad/sec. $\rightarrow f_1 = 540$ cps

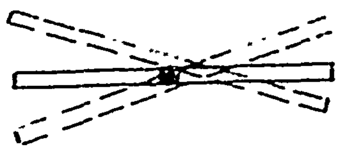
$P_{II} = 4890$ rad/sec. $\rightarrow f_2 = 780$ cps

the corresponding amplitude ratios amd mode shapes from Equation A-2

for P_I

amplitude ratio $$\alpha_I = \frac{P_I^2 - \frac{k_x}{m}}{\frac{k_x R}{m}} = -1.94 = \frac{\theta}{x} \quad \therefore \frac{x}{\theta} = -.515$$

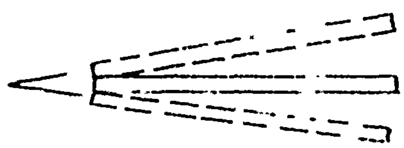
mode shape



for P_{II}

amplitude ratio $$\alpha_{II} = \frac{P_{II}^2 - \frac{k_x}{m}}{\frac{k_x R}{m}} = .335 = \frac{\theta}{x} \quad \therefore \frac{x}{\theta} = 2.96$$

mode shape



The first mode was observed in the laboratory at 560 cps with the mode shape being essentially as calculated for P_I. In this mode, the vial amd scale tend to rotate about a point near the mortar tube.

The complete solution of the differential equations for the free vibration era is:

$$x = A_1 \cos P_I t + A_2 \sin P_I t + A_3 \cos P_{II} t + A_4 \sin P_{II} t$$

$$\theta = \alpha_I A_1 \cos P_I t + \alpha_I A_2 \sin P_I t + \alpha_{II} A_3 \cos P_{II} t + \alpha_{II} A_4 \sin P_{II} t \qquad \text{(A-5)}$$

To evaluate the nature of the module's response motions during weapon firing, assume a pure translational movement of the handle resulting from a sinusoidal acceleration

$$\ddot{X}_g(t) = G \sin \omega t$$

where G is the magnitude of acceleration at frequency ω rad/sec. The corresponding velocity and displacement of the foundation then are

$$\text{velocity} \qquad \dot{X}_g = \frac{G}{\omega}(1 - \cos \omega t)$$

$$\text{displacement} \qquad X_g = \frac{G}{\omega}\left(t - \frac{\sin \omega t}{\omega}\right)$$

During the time of the foundation movement, the differential equations are

$$m\ddot{x} + k_x(x + R\theta) = \frac{k_x G}{\omega}\left(t - \frac{\sin \omega t}{\omega}\right)$$

$$J\ddot{\theta} + (k_\theta + k_x R^2)\,\theta + k_x R X = \frac{RG}{\omega}\left(t - \frac{\sin \omega t}{\omega}\right)$$

For solution assume

$$x = N_1 \sin \omega t + N_3 t \qquad \theta = N_2 \sin \omega t + N_4 t \qquad \text{(A-6)}$$

Substituting into the differential equations and solving

$$N_1 = \frac{\frac{G}{\omega^2}\left(-\frac{k_x k_\theta}{mj} + \frac{k_x}{m} w^2\right)}{\omega^4 - \omega^2\left(\frac{k_x}{m} + \frac{k_\theta + k_x R^2}{j}\right) + \frac{k_x k_\theta}{mJ}} = \frac{\frac{G}{\omega^2}\left(-\frac{k_x k_\theta}{mJ} + \frac{k_x}{m}\omega^2\right)}{(\omega^2 - P_I^2)(\omega^2 - P_{II}^2)}$$

$$N_2 = \frac{G\frac{k_x R}{J}}{\omega^4 - \omega^2\left(\frac{k_x}{m} + \frac{k_\theta + k_x R^2}{J}\right) + \frac{k_x k_\theta}{mJ}} = \frac{G\frac{k_x R}{J}}{(\omega^2 - P_I^2)(\omega^2 - P_{II}^2)}$$

$$N_4 = 0 \qquad N_3 = \frac{G}{\omega} \qquad \text{(A-7)}$$

The complete solution of the differential equations is

$$x = A_1 \cos P_I t + A_2 \sin P_I t + A_4 \sin P_{II} t + N_1 \sin \omega t + \frac{G}{\omega} t$$

$$\theta = \alpha_I A_I \cos P_I t + \alpha_I A_2 \sin P_I t + \alpha_{II} A_3 \cos P_{II} t + \alpha_{II} A_4 \sin P_{II} t + N_2 \sin \omega t \qquad \text{(A-8)}$$

from the initial conditions

$$@\, t = 0, \qquad x = \theta = 0$$

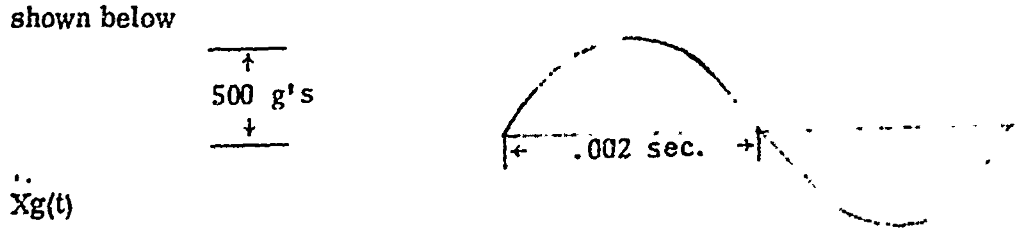
$$\because \quad A_1 = A_3 = 0$$

$$@\, t = 0, \qquad \dot{x} = \dot{\theta} = 0$$

$$\because \quad 0 = A_2 P_I + P_{II} + N_1 \omega + \frac{G}{\omega}$$

$$0 = \alpha_I A_2 P_I + \alpha_{II} A_4 P_{II} + N_2 \omega \qquad \text{(A-9)}$$

All constants can be evaluated for a specific motion of the mortar Xg(t). From Reference 1 it seems reasonable to represent the motion of the weapon with a sinusoidal acceleration of about 500 g's and half period of 2 ms. as shown below



$$G = 500 \text{ g's} = 500\ (386) \text{ in/Sec}^2 \qquad \omega = \frac{2\Pi}{.004} = 1570 \text{ rad/sec.}$$

[1] L. Heppner, "Special Study of Setback and Spin for Artillery and Tank Ammunition," April 1966, Aberdeen Proving Ground, Report No. DPS-1963.

from Equation A-7 then

$N_1 = .08924 \qquad N_2 = .004$

and from Equation A-9

$A_4 = .00243 \qquad A_2 = .00157$

The equations for the system's displacement during the shock then are

$$x = .00157 \text{ Sin } 3370t + .00243 \text{ Sin } 4890t - .08924 \text{ Sin } 1570t + 122.93t$$

$$\theta = -.003046 \text{ Sin } 3370t + .00081 \text{ Sin } 4890t + .004 \text{ Sin } 1570t$$

The corresponding equations for acceleration then are

$$x = -17830 \text{ Sin } 3370t - 58100 \text{ Sin } 4890t + 220,000 \text{ Sin } 1570t$$

$$\theta = 34600 \text{ Sin } 3370t - 19400 \text{ Sin } 4890t - 9860 \text{ Sin } 1570t$$

The equations for the relative displacement and acceleration time histories are given in Figures A-1 and A-2.

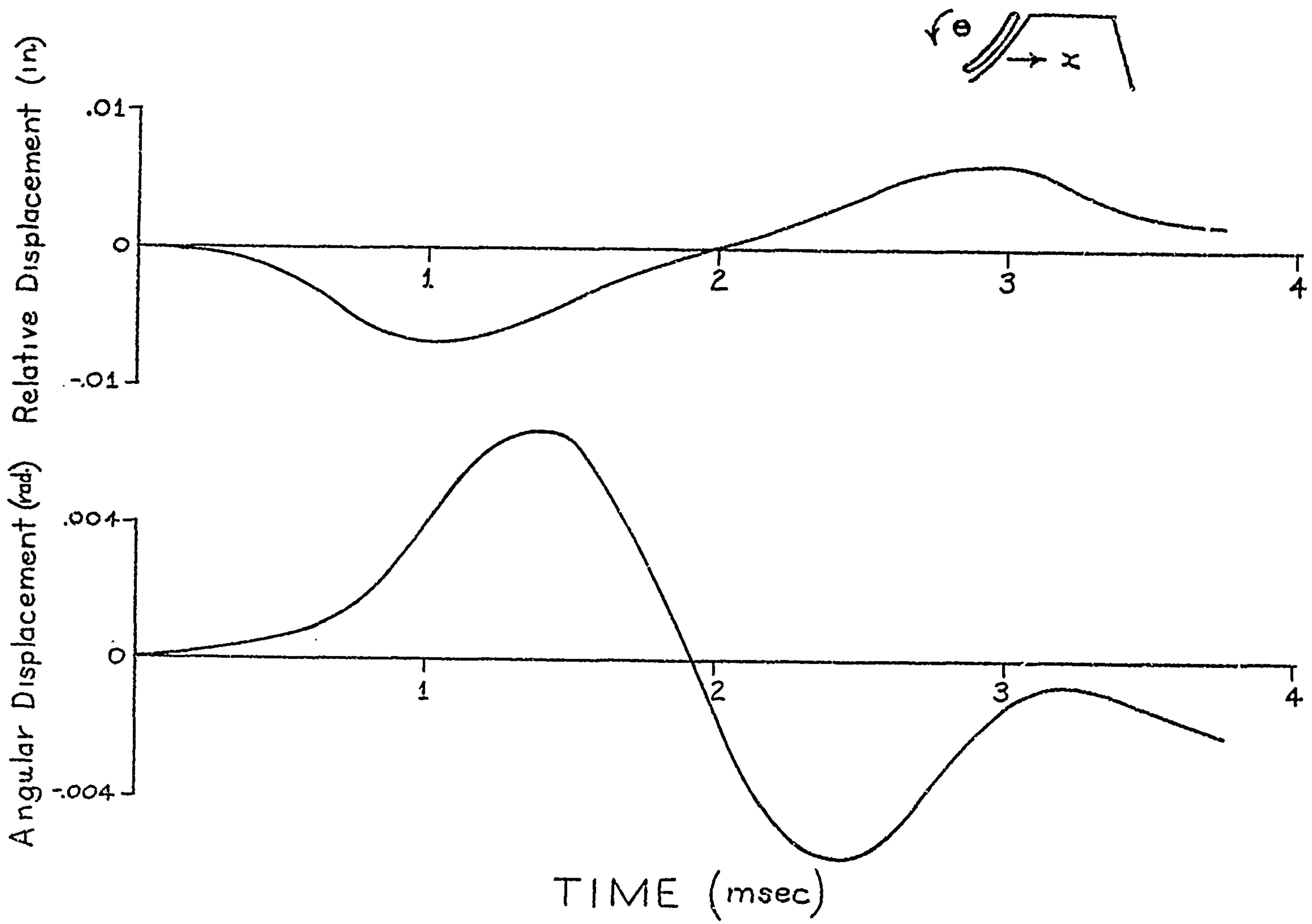


Figure A-1. Response of Elevation Indicator Module to Shock Excitation - Displacement Time History

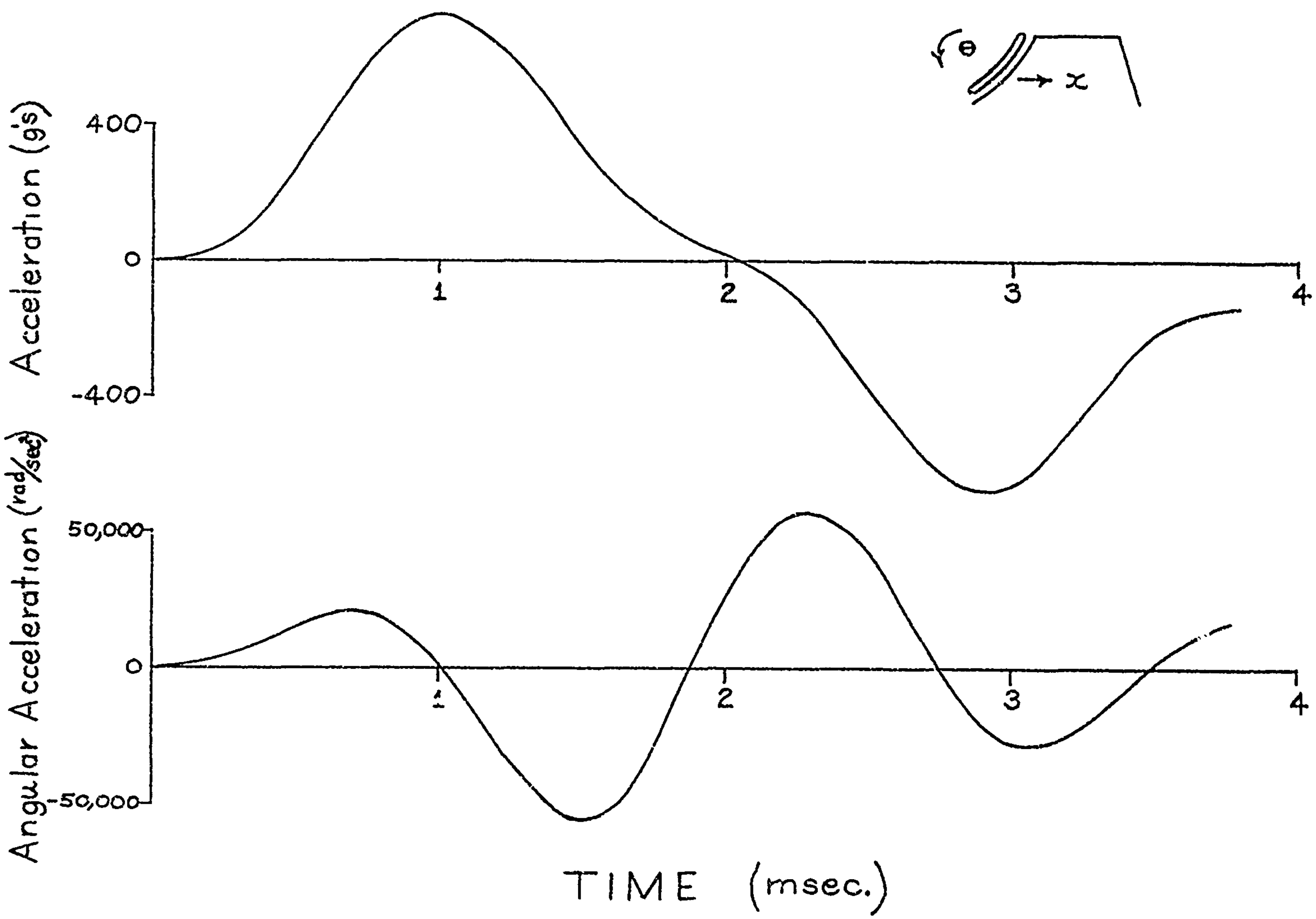


Figure A-2. Response of Elevation Indicator Module to Shock Excitation - Acceleration Time History